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Pakistan Institute of Development Economics

2005

The Pakistan Development Review
44 : 2 (Summer 2005) pp. 117–129

P-Star Model: A Leading Indicator of Inflation for Pakistan

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The P-star inflation model is based on the long-term quantity theory of money and puts together the long-term determinants of the price level and the short-run changes in current inflation. The P-star model-based indicator has replaced the previous monetary policy procedures in a number of countries because it offers by far more information and predictive power than monitoring movements in money supply and the rate of monetary growth. In this paper we used the P-star model to calculate the leading indicator of inflation, and also to test the forecasting performance of the P-star model-based leading indicator of inflation. The results of the study show that compared to the simple autoregressive model and the M2 growth augmented model, the P-star model can be used to obtain the leading indicator of inflation in Pakistan because it has additional information about the future rate of inflation. Therefore, this paper provides a useful tool to the policy-makers to assess the future movement of inflation in Pakistan.

1. INTRODUCTION

Forecasting based on leading indicators has a long history in economics. Much of the earlier work concentrated on developing indicators of macro-economic variables such as inflation [Clements and Hendry (1998)]. Since the emergence of the phenomenon of 'missing money' in the 1970s, the stability and predictability of the rate of inflation has emerged as one of the main objectives of the monetary policy the world over. Therefore, a number of central banks, for example New Zealand (1990), Canada (1991), UK (1992), Sweden (1993), Finland (1993), Australia (1993), Spain (1994) and Czechoslovakia (1998), among others, have changed their previous monetary policy procedures and shifted to publicly announced inflation targeting in the 1990s, using the P-star model as an indicator of inflation [Hallman, *et al.* (1991)]. The P-star model-based indicator offers by far more information and predictive power than monitoring increments in money supply and the rate of monetary growth, as under the previous procedures.

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Since it is defined as the money per unit of real potential output, deviations between the actual price (P) and P-star, the price gap, indicates future acceleration or deceleration of inflation, provided P and P-star are cointegrated. While in all the standard models of inflation the output gap is a major explanatory variable for inflation, in the P-star approach, the deviations of the velocity of money from "trend" levels also matter for price level determination.

The P-star inflation model is based on the long-term quantity theory of money and puts together the long-term determinants of the price level and the short-term changes in current inflation. In this paper we intend to use this model to identify the long-run equilibrium price level as a variable determined by current money supply, potential income, and the equilibrium rate of money circulation. This will be followed by tracking forecasting performance of the P-star model based leading indicator of inflation.

In the literature the reactions to the forecasting performance of the P-star model are mixed. While Hallman, *et al.* (1989, 1991) and Christano (1989) show that the P-star performs better than other models, Pecchenino and Rasche (1990) find that the P-star model implies unreasonable dynamic behaviour. Hoeller (1991) in his study of the P-star model on all OECD countries reports that the results of the model were not impressive in the case small OECD countries. However, Kool and Tatom (1994) attribute such results for the smaller countries to the fact that the smaller countries tend to import inflation, and when adjusted for this factor they report improvement in results.

The outline of the paper is as follows: we shall explain the P-star model of inflation indicator, discuss methodology to measure potential output and trend velocity and data used to estimate the model in Section 2; leading indicator of inflation in Pakistan is estimated in Section 3; the regression results and causality analysis are presented in Section 4, followed by a comparison of the tracking and forecasting performance of the model in Section 5. Section 6 concludes the paper.

2. THE P-STAR MODEL OF INFLATION

Following Hallman, *et al.* (1989, 1991) we develop the P-star model of inflation based on the famous equation of exchange in the family of quantity theory of money, i.e.,

$$PY = MV \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

Where P is the price level, M is a monetary aggregate, V the income velocity of money and Y is output at constant prices. This model links the behaviour of price level to the growth of money supply depending on two basic propositions; that the real output fluctuates around potential real output (Y^*), and the income velocity of money has equilibrium level (V^*). By using long run equilibrium values of real output and velocity we can obtain equilibrium level of aggregate price level, P^* by the following identity;

$$P^* = M \times V^* / Y^* \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Taking logs on the both sides we can rewrite Equation 2 as:

$$p^* = (m + v^* - y^*) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

In the theory it is assumed that actual price level (p) tends to move towards the equilibrium price level (p^*). The p-star model postulates that the difference between the actual and long run equilibrium price level acts as a good predictor of inflation. The leading indicator of inflation in this study is defined as:

$$\pi^* = p - p^* = (v - v^*) - (y - y^*) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Therefore this model can be used to directly predict movements of the rate of inflation. It implies that if actual inflation (π) exceeds the predicted inflation by this model (π^*), then P-star model predicts that the inflation will fall in future until it reaches the equilibrium rate (π^*) and vice versa.

The price gap however does not contain information about the dynamics of adjustment of p to p^*. Therefore, in this paper, an error correction model of the adjustment process is adopted and the general dynamic specification of the model is given by Equation 5 as:

$$\Delta p = \alpha_0 + \alpha_1 (p_{t-1} - p^*_{t-1}) + \sum_{i=1_1}^{n} \beta_i \Delta p_{t-i} + \varepsilon_t \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

The coefficient α_1 is the speed of adjustment of prices to P^* and the coefficients of β_i represent the lag of the actual rate of inflation.

The critical issue in this model is the estimation of potential output and the equilibrium values of income velocity of money. A number of techniques available to obtain the value of potential output can be categorised into two broad groups; the economic theory based approach and the statistical approaches. Braun (1990) used the economic theory based approach to derive the value of potential output by combining a Phillip's curve based estimate of the natural rate of unemployment with Okun's law. These estimates are also adopted by Ebrill and Fries (1990) and Pacchenino and Rasche (1990). Ebrill and Fries (1990) calculate the velocity gap as the residuals from a co-integrating equation explaining long-run velocity by the own and competing rates of return on M2. Christiano (1989), Hannah and James (1989) and Hallman, *et al.* (1991) used a linear time trend to calculate potential output. Among the statistical approaches Bomhoff (1990), Kuttner (1992), and Fisher and Fleissing (1995) used the Kalman Filter, whereas Hoeller and Paret (1991), Gibbs (1995), and MeMorrow and Roeger (2001), among others, used Hodrick-Prescort filter approach [Hodrick and Prescort (1980)]. This study also uses the widely applied Hodrick-Prescort filter approach to estimate equilibrium output and velocity. This method basically uses a long run symmetric moving average to de-trend the particular series. Technically the HP filter is a two-sided linear filter i.e.,

$$\text{Minimise } \sum_{t=1}^{T}\left(ln Y - ln Y_t^*\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (6)$$

the sum of the squared deviations of a variable (in this case, output), Y_t, from its trend

$$\text{Subject to } \sum_{t=2}^{T-1}\left[\left(ln Y_{t+1} - ln Y_t^*\right) - \left(ln Y_t^* - ln Y_{t-1}\right)\right]^2 \le e$$

HP method chooses lnY^* to

$$\text{Minimise } \sum_{t=1}^{T}\left(ln Y - ln Y_t^*\right)^2 + \lambda \sum_{t=2}^{T-1}\left[\left(ln Y_{t+1} - ln Y_t^*\right) - \left(ln Y_t^* - ln Y_{t-1}\right)\right]^2 \qquad (7)$$

Where Y_t is the actual GDP at constant market prices, Y_t^* is the trend GDP at constant market prices, and the λ is Lagrange multiplier. The λ may be termed as penalty parameter that controls the smoothness of the series variance. It implies that the larger the value of λ, the smoother the series: as $\lambda \to \infty$, lnY_t^* approaches a linear trend. In terms of output gaps a smaller λ implies shorter cycles and smaller gaps. Following what has become the norm in the literature and among practitioners, this paper sets λ at 100.

The study covers data period 1960 to 2003, and the two main data sources are the Federal Bureau of Statistics of Pakistan (Various Issues) and the International Financial Statistics (2004).

3. ESTIMATED INDICATOR OF INFLATION

One of the important assumptions of the model by Hallman, *et al.* (1989, 1991) is that the velocity of money is stationary and the long-run measure of velocity can be obtained by a simple average. However, this assumption does not hold as shown by the outcome of the Augmented Dickey-Fuller test of unit root in Table 1. The results show that velocity is not stationary and that implies that we cannot get the equilibrium value of velocity of money by the simple average. Therefore, this study adopts the widely recommended Hodrick-Prescort filter approach to calculate the equilibrium value of velocity (V^*).

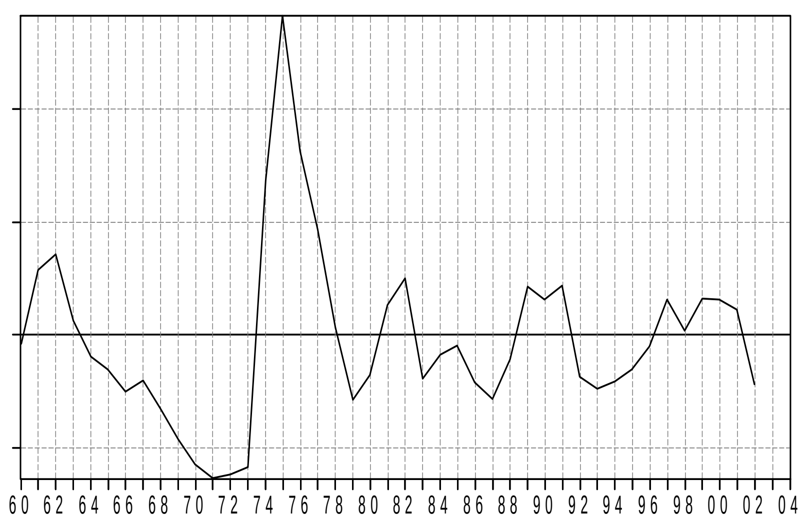
The estimated price gap π^* (that is $p–p^*$) as an indicator of inflation is presented in the Figure 1. It shows that the gap between the actual prices and the P-star model predicted equilibrium prices (P^*) remains positive during the early

Table 1

Augmented Dickey-Fuller Test Statistic

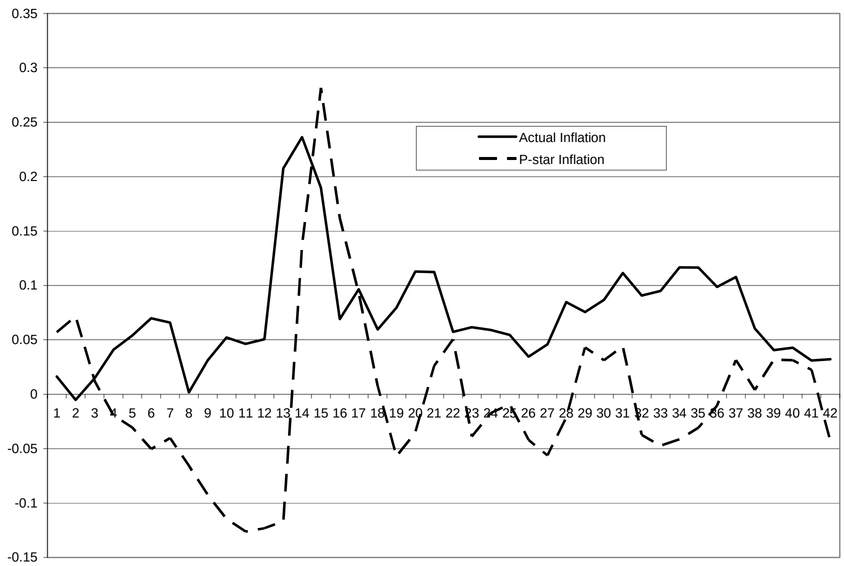
Variables	Test Statistics	Lag	*t*-value	*P*-value
LV	Constant	1	–1.978088	0.2950
Δ LV	None	0	–5.047787	0.0000
LP-star	Constant, Trend	0	–3.221457	0.0941
Δ LP-star	Constant	0	–5.355803	0.0001
LP	Constant, Trend	1	–3.412787	0.0636
Δ LP	Constant	1	–3.522934	0.0124
P-star Inflation	None	1	–4.431786	0.0000

Fig. 1. Gap between the Actual Prices and the Equilibrium Prices (P*).



sixties, mid seventies, early eighties, early and late nineties, and early years of 2000. It also reveals that actual prices are less than the model predicted prices for the last two years. This implies that in future actual prices would move upward towards equilibrium prices, as shown in Figure 2.

Fig. 2. The Actual Inflation and the P-star Indicator of Inflation.



4. COINTEGRATION AND CAUSALITY ANALYSIS

In this section we estimate the long run as well as dynamic relationship and direction of Granger causality between the actual rate of inflation and the P-star based leading indicator of inflation. The objective is to test whether P-star model can be used to calculate a leading indicator of inflation in Pakistan or not.

Implicit assumption of the theory is that there is a long run relationship between the actual prices and equilibrium prices [Hallman, *et al.* (1991)] and it is also assumed that there is a one to one relationship between both prices. We test these propositions in this section. To test the existence of cointegrating relationship between the actual and equilibrium prices we used the Engle-Granger (1987) two step method written as;

$$LP_t = \alpha + \beta\, LP^*_t + \varepsilon_t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (8a)$$

$$\Delta\varepsilon_t = \rho\varepsilon_{t-1} + \beta_1\Delta\varepsilon_{t-1} + \beta_2\Delta\varepsilon_{t-2} + \ldots + \beta_p\Delta\varepsilon_{t-p} + \mu_t \quad \ldots \quad \ldots \quad \ldots \qquad (8b)$$

where LP is log of price index, LP^* is log of equilibrium prices calculated in the previous section, ε_t is the residual from cointegrating equation and μ_t is the residual from the Equation 8b of the ADF unit root test which is assumed to be white noise. The results of the long run Equation 8a—the first step of Engle-Granger method—are given by Equation 9.

$$\underset{(87.039)}{LP = 1.0025\, LP^*} \underset{(-0.290)}{- 0.0118} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (9)$$

R-squared 0.994617 Durbin-Watson stat 0.655984
Augmented Dickey-Fuller test statistic –4.477

Figures in the parentheses show t-statistics.

The second step of the Engle-Granger procedure is to test the hypothesis of unit root in the residual obtained from Equation 9 by applying the ADF test. If the residual term has no unit roots, i.e., it is I(0), then we can conclude that both variables are cointegrated. The results from the ADF test statistics show that the residual term is stationary and that implies that there is long run relationship between the actual prices and the equilibrium prices calculated by the P-star model. The estimated parameter of the equilibrium price level is close to one. We formally tested that the estimated parameter is one by applying Wald test and the results show that there is one to one relationship between the actual and P-star prices.

The Granger representation theorem states that if there exists a cointegrating relationship between the two variables then there exists at least one-way Granger causality between them. This theorem further implies that if the two series are non-stationary and they have cointegrating relationship between them then the dynamic system can be represented by equilibrium correcting mechanism. In the following we estimate the dynamic relationship by specifying the equilibrium correcting mechanism (EqCM) and test Granger causality between the variables.

The dynamic equilibrium correcting model is estimated by applying general to specific methodology. The results of the preferred dynamic model for inflation prediction are given by Equation 10.

$$INF_t = \underset{(16.28)}{0.9395\, INF_{t-1}} \underset{(-4.12)}{- 0.275\, EqCM_{t-1}} \quad \ldots \quad \ldots \quad \ldots \qquad (10)$$

R-squared	0.573	Durbin-Watson stat	2.146
LM Test: $\chi^2_{(1)}$	0.00	ARCH Test: $\chi^2_{(1)}$	0.150

Figures in the parentheses show t-statistics.

It shows that the residual passed all the required diagnostic tests and the estimated coefficients have *a priori* expected signs. The estimated coefficient of $EqCM_{t-1}$ term indicates a speed of adjustment of the rate of inflation towards the equilibrium state. It implies that economic agents correct approximately 28 percent of their errors during one year. The significance of error correction term in the equation also implies that P-star inflation indicator causes the actual rate of inflation. More precisely in the words of Granger and others, the P-star indicator of inflation predicts the future rate of inflation. From this we can infer that the higher the actual prices from the equilibrium prices today implies low rate of inflation in the future.

The cointegration analysis and the results of the error correcting model indicate that there is a causality between the actual inflation and the P-star model based inflation indicator. The important question is whether P-star model based measure of inflation can be used as leading indicator of inflation for forecasting. In order to decide the sequence of causality (prediction), whether it is unidirectional or bidirectional, pair wise Granger Causality test is applied. The hypothesis that P-star model calculated indicator of rate of inflation does not predict the actual rate of inflation is rejected at five percent level of significance. The F-values for Granger bivariate causality test is 2.774. When we test for the inverse causality that is inflation does not Granger causes the P-star inflation indicator, the hypothesis is accepted implying inflation does not predict the indicator. On the basis of analysis we can conclude that there is unidirectional causality that runs from the P-star inflation indicator to the actual rate of inflation.

5. FORECASTING PERFORMANCE OF THE P-STAR INDICATOR OF INFLATION

Since the main objective of this paper is to develop a leading indicator of inflation by using P-star model, we performed a forecasting exercise using the univariate autoregressive model as the benchmark model. Moreover to compare the forecasting performance of preferred leading indicator we used money (M2) growth as another leading indicator of inflation. Currently State Bank of Pakistan is using money growth as one of the indicators of future inflation. The performance of the P-star based leading indicator is evaluated with the simulation of out of the sample forecasting. To get the forecasted value of inflation in Pakistan we estimated the following equation:

$$\pi_{t+h} = \alpha_0 + \alpha_1\, \pi_t^* + \sum_{i=1_1}^{n} \beta_i\, \pi_{t-i} + \varepsilon_{t+h} \qquad \ldots \quad \ldots \quad \ldots \qquad (11)$$

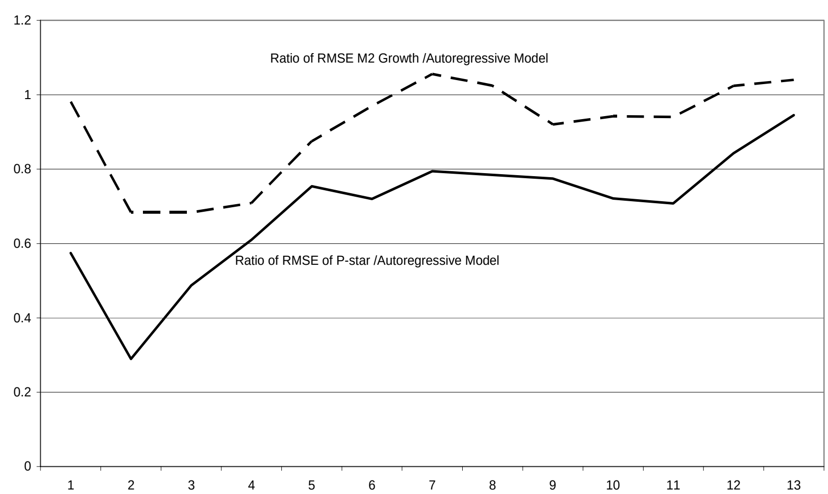
Where π_{t+h} is the h-period ahead inflation and π_t^* is an indicator of inflation whose forecasting performance is being evaluated. The data used for this study, as mentioned earlier, spans from 1960 to 2003. Out of sample forecasts are made from 1990 to 2003 (detail results are given in the Table 2).[1]

[1]The forecasting performance of the P-star model-based leading indicator of inflation did not improve significantly by considering the 1973 inflation as an outlier.

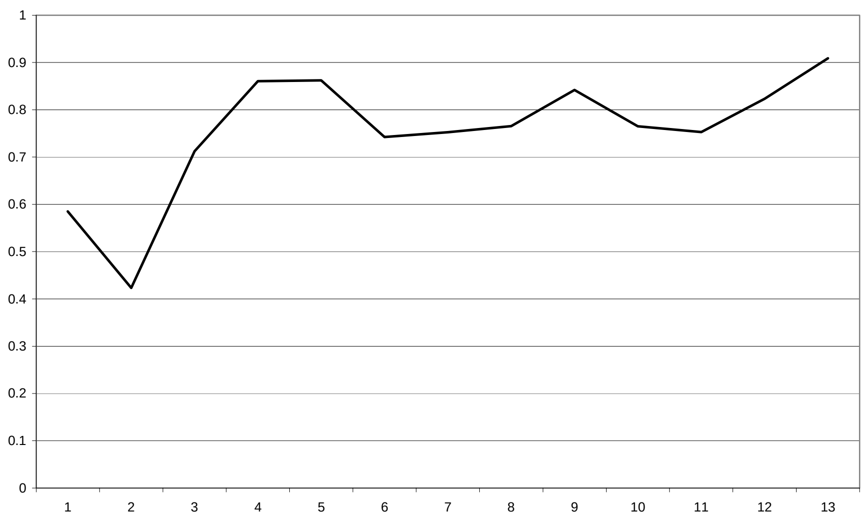
Table 2

The forecasting performance is evaluated by the Root Mean Square Error (RMSE) and the relative RMSE to a simple univariate autoregressive model. The reduction in RMSE and less than one value of the ratio of the leading indicator's RMSE corresponds with the benchmark model's RMSE indicates improvement in the forecasting by using leading indicator [Stock and Watson (1999)]. The exercise was performed for forecasts of inflation from 1 to 13 years ahead. The results from recursive estimation and forecasting performance of autoregressive model, M2 growth augmented model and the P-star indicator model are presented in the Table 2. The fourth column of Table 2 reports the Root Mean Square Error (RMSE) of the univariate autoregressive model and the fifth column reports forecasting performance of growth in M2 as an indicator of inflation.

As can be seen from Table 2, the RMSE of M2 growth augmented model is decreased relative to autoregressive model for 1 to 6 years ahead forecasting period. It means that for short and medium forecasting time horizon the M2 growth indicator has additional information about future rate of inflation than the simple model. However, the forecasting performance of P-star model indicator shows that in forecasting inflation the P-star indicator augmented model performs better than the univariate autoregressive model. The RMSE of P-star indicator augmented model is less than the RMSE of autoregressive model. The ratio RMSE of P-star indicator and M2 growth indicator with RMSE of univariate autoregressive are also calculated and plotted in Figure 3.



We also compare the forecasting performance of the M2 growth indicator augmented model and the P-star indicator augmented model by calculating the ratio of the RMSE of M2 growth model to the P-star model. If the value of this ratio is less than one it indicates better performance. As may be seen from the Table 2 and Figure 4, P-star based indicator of inflation performs better than M2 growth indicator to forecast future inflation. Thus we can safely conclude that the indicator calculated by the P-star model can be used as a leading indicator of inflation in Pakistan.



6. CONCLUSIONS AND POLICY IMPLICATIONS

Forecasting based on leading indicators has a long history in economics. Much of the earlier work concentrated on developing indicators of macroeconomic variables such as inflation. While forecasting on the basis of leading indicators is also emerging fastly. One of the leading indicators of inflation is based on the P-star model. The P-star inflation model is based on the long-term quantity theory of money and puts together the long-term determinants of the price level and the short-term changes in current inflation. In this paper we used the P-star model to calculate the leading indicator of inflation and also tested the forecasting performance of P-star model based leading indicator of inflation.

The results of the study show quite clearly that compared to the simple autoregressive model and M2 growth augmented model the P-star model can be used to obtain the leading indicator of inflation in Pakistan because it has additional information about the future rate of inflation. Therefore, this paper provides a useful tool to the policy-maker to assess the future movement of inflation in Pakistan.

Finally, in future the research can be extended in following directions, firstly by using high frequency data i.e., quarterly or monthly, and secondly by including other indicators of inflation such as Philips curve, interest rate spread and credit growth and compare the forecasting performance.

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Table 2

Forecasting Performance of the P-Star Indicator of Inflation

Estimation Period	Forecasting Period	h-step	Root Mean Square Error			Ratio of RMSE		
			Autoregres-sive	M2 Growth	P-star indicator	M2 / Auto	P-star / Auto	P-star / M2
1960–1990	1990–2003	13	0.020	0.0208	0.0189	1.04	0.945	0.90865
1960–1991	1991–2003	12	0.021	0.0215	0.0177	1.023809	0.842857	0.82325
1960–1992	1992–2003	11	0.0202	0.0190	0.0143	0.940594	0.707920	0.75263
1960–1993	1993–2003	10	0.0208	0.0196	0.0150	0.942307	0.721153	0.76530
1960–1994	1994–2003	9	0.0213	0.0196	0.0234	0.920187	0.774647	0.84183
1960–1995	1995–2003	8	0.0204	0.0209	0.0160	1.024509	0.784313	0.76555
1960–1996	1996–2003	7	0.0214	0.0226	0.0170	1.056074	0.794392	0.75221
1960–1997	1997–2003	6	0.0232	0.0225	0.0167	0.969827	0.719827	0.74222
1960–1998	1998–2003	5	0.0232	0.0203	0.0175	0.875	0.754310	0.86206
1960–1999	1999–2003	4	0.0182	0.0129	0.0111	0.708791	0.609890	0.86046
1960–2000	2000–2003	3	0.0193	0.0132	0.0094	0.683937	0.487046	0.71212
1960–2001	2001–2003	2	0.0228	0.0156	0.0066	0.684210	0.289473	0.42307
1960–2002	2002-2003	1	0.0162	0.0159	0.0093	0.981481	0.574074	0.58490